

# Open M-branes on $\text{AdS}_{4/7} \times \text{S}^{7/4}$ Revisited

Makoto Sakaguchi<sup>†a</sup> and Kentaroh Yoshida<sup>\*b</sup>

<sup>†</sup> *Osaka City University Advanced Mathematical Institute (OCAMI),  
Sumiyoshi, Osaka 558-8585, Japan.*

<sup>\*</sup> *Theory Division, High Energy Accelerator Research Organization (KEK),  
Tsukuba, Ibaraki 305-0801, Japan.*

<sup>a</sup>msakaguc@sci.osaka-cu.ac.jp      <sup>b</sup>kyoshida@post.kek.jp

## Abstract

We proceed further with a study of open supermembrane on the  $\text{AdS}_{4/7} \times S^{7/4}$  backgrounds. Open supermembrane can have M5-brane and 9-brane as Dirichlet branes. In AdS and pp-wave cases the configurations of Dirichlet branes are restricted. A classification of possible Dirichlet branes, which is given up to and including the fourth order of fermionic variable  $\theta$  in hep-th/0310035, is shown to be valid even at full order of  $\theta$ . We also discuss open M5-brane on the  $\text{AdS}_{4/7} \times S^{7/4}$ .

**Keywords:** Open membrane, M5-branes, Penrose limit, pp-wave

# 1 Introduction

Supermembrane [1,2] plays a fundamental role in a promising formulation of M-theory [3]. The discrete light-cone quantized M-theory is considered to be described by a matrix model [3], which is also obtained by dimensionally reducing the ten-dimensional super Yang-Mills theory to one dimension. On the other hand, an open supermembrane was considered in [4] and [5]. It has Dirichlet  $p$ -branes for  $p = 1, 5$  and  $9$  like D-branes in superstring theories. The  $p = 5$  and  $p = 9$  cases correspond to an M5-brane and the end-of-the world 9-brane in the Horava-Witten theory [6], respectively. An M2-brane ending on an M5-brane on general supergravity backgrounds was also discussed within the context of the superembedding in the work of Chu and Sezgin [7], where the  $\kappa$ -invariance of an open M2-brane may govern the dynamics of M5-branes.

The light-cone gauge has been used for most of those studies of strings, membranes and D-branes on pp-waves, and so the analyses of them are not covariant. A light-cone analysis of an open supermembrane\* on the maximally supersymmetric pp-wave background in eleven dimensions was also carried out in [11]. A covariant approach to study D-branes in flat space was proposed by Lambert and West [14]. The covariant approach can be applied to the type IIB strings [15,16] and the type IIA strings [17] on pp-waves, which are given by Bain-Peeters-Zamaklar [18] and Hyun-Park-Shin [19], respectively. The  $S^1$  compactification of M5-brane and 9-brane attached to an open supermembrane gives D4-brane and D8-brane, respectively, allowed in type IIA pp-wave background [17]. D-branes on a type IIA pp-wave background are intensively studied in [20]. The method of [14] is applicable to classify possible configurations of  $1/2$  supersymmetric (SUSY) Dirichlet branes of an open supermembrane on the pp-wave [21]. The 9-brane coupled to an M2-brane on the pp-wave was considered in a study of a heterotic plane-wave matrix model [22].

The covariant approach is also applicable to the AdS backgrounds, as well as pp-waves. In fact, we have extended the work [21] to supermembranes on the  $\text{AdS}_{4/7} \times S^{7/4}$  backgrounds [23] in our work [24]. As a matter of course, the results in the AdS cases are related to those in the pp-wave through the Penrose limit [25,26] (For Penrose limit of superalgebra, see [27]).

---

\*Matrix model on the eleven-dimensional pp-wave background [8] was proposed by Berenstein-Maldacena-Nastase [9] and the spectrum of this matrix model was studied in [10]. In [11,12] supermembrane theory on the pp-waves which is related to the matrix model via the matrix regularization [2], is discussed from several viewpoints. For a relation to the eleven-dimensional supergravity, see [13].

Notably, the classifications of 1/2 SUSY Dirichlet branes agree with the results of the brane probe analysis given by Kim and Yee [28]. Similarly, we can discuss D-branes of covariant AdS-superstring [29]. The Penrose limit of the classification obtained in [29] recovers that in [18]. These results are also consistent to the brane probe analysis given by Skenderis and Taylor [30].

Here we should recall that the classifications of possible configurations of 1/2 SUSY Dirichlet branes of an open supermembrane in AdS and pp-wave have been given by the fourth order analysis of a fermionic variable  $\theta$ . In this paper we show that our classification result is still valid even at full order of  $\theta$ . This proof makes the classification of 1/2 SUSY Dirichlet branes complete. This proof follows our previous work [31], where the full order proof was given in the case of AdS-string. We also discuss an open M5-brane on the  $\text{AdS}_{4/7} \times S^{7/4}$  by using the covariant Pasti-Sorokin-Tonin (PST) [32] type action proposed by Claus [33] and following our strategy. As a result, we see that no 1/2 SUSY Dirichlet brane of an open M5-brane on the  $\text{AdS}_{4/7} \times S^{7/4}$  is allowed. This is the case even in flat space.

This paper is organized as follows: Section 2 is devoted to a setup for our consideration later. We introduce an open supermembrane on the  $\text{AdS}_{4/7} \times S^{7/4}$  and the classification of 1/2 SUSY Dirichlet branes at the fourth order of  $\theta$  is summarized. In section 3, we show that the classification reviewed in section 2 is still valid at full order of  $\theta$ . It is also shown that the classification of 1/2 SUSY Dirichlet branes in the pp-wave case holds at full order of  $\theta$ . In section 4, we discuss the  $\kappa$ -invariance of an open M5-brane on the  $\text{AdS}_{4/7} \times S^{7/4}$ . We see that no 1/2 SUSY Dirichlet brane is allowed to exist. Section 5 is devoted to a conclusion and discussion.

## 2 Dirichlet Branes of Open Supermembrane on $\text{AdS}_{4/7} \times S^{7/4}$

In this section we will introduce the action of an open supermembrane on the  $\text{AdS}_{4/7} \times S^{7/4}$  [23] and review the classification of 1/2 SUSY Dirichlet branes [24].

### 2.1 Covariant Action of M2-brane on $\text{AdS}_{4/7} \times S^{7/4}$

The supermembrane action [1, 2] on the  $\text{AdS}_{4/7} \times S^{7/4}$  backgrounds was proposed by de Wit-Peeters-Plefka [23]. It consists of the Nambu-Goto part and the Wess-Zumino term:

$$\mathcal{L} = \mathcal{L}_{\text{NG}} + \mathcal{L}_{\text{WZ}}. \quad (2.1)$$

The Nambu-Goto part is represented, using the induced metric  $g_{ij}$  on the world-volume of membrane, by

$$\mathcal{L}_{\text{NG}} = -\sqrt{-\det g_{ij}}, \quad g_{ij} = E_i^M E_j^N G_{MN} = E_i^A E_j^B \eta_{AB}, \quad (2.2)$$

where  $G_{MN}$  is a target space metric and  $Z^{\widehat{M}} = (X^M, \theta^\alpha)$  is a target superspace coordinate. We have also introduced the pull-back supervielbein  $E_i^A \equiv \partial_i Z^{\widehat{M}} E_{\widehat{M}}^A$ . The Wess-Zumino term consists of two parts as  $\mathcal{L}_{\text{WZ}} = \mathcal{L}_{\text{WZ}}^{\text{bose}} + \mathcal{L}_{\text{WZ}}^0$  and each part is given by, respectively,

$$\mathcal{L}_{\text{WZ}}^{\text{bose}} = \frac{1}{6} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} C_{A_1 A_2 A_3}, \quad (2.3)$$

$$\mathcal{L}_{\text{WZ}}^0 = i \int_0^1 dt \bar{\theta} \Gamma_{AB} E(X, t\theta) \wedge E^A(X, t\theta) \wedge E^B(X, t\theta). \quad (2.4)$$

Supervielbeins on the  $\text{AdS}_{4/7} \times S^{7/4}$  can be obtained via the coset construction with the coset supermanifolds:

$$\text{AdS}_4 \times S^7 \sim \frac{OSp(8|4)}{SO(1,3) \times SO(7)}, \quad \text{AdS}_7 \times S^4 \sim \frac{OSp(2,6|4)}{SO(1,6) \times SO(4)}. \quad (2.5)$$

Parameterizing the manifolds as  $g(X, \theta) = e^{PX} e^{\theta Q}$ , we obtain the expression of supervielbeins:

$$E^A = dX^M e_M^A - i \bar{\theta} \Gamma^A \left( \frac{2}{\mathcal{M}} \sinh \frac{\mathcal{M}}{2} \right)^2 D\theta, \quad E^{\bar{\alpha}} = \left( \frac{\sinh \mathcal{M}}{\mathcal{M}} D\theta \right)^{\bar{\alpha}}, \quad (2.6)$$

where we have introduced the following quantities:

$$\begin{aligned} (D\theta)^{\bar{\alpha}} &\equiv d\theta^{\bar{\alpha}} + e^A (T_A^{B_1 \dots B_4} \theta)^{\bar{\alpha}} F_{B_1 \dots B_4} - \frac{1}{4} \omega^{A_1 A_2} (\Gamma_{A_1 A_2} \theta)^{\bar{\alpha}}, \\ i\mathcal{M}^2 &= 2(T_A^{B_1 \dots B_4} \theta) F_{B_1 \dots B_4} (\bar{\theta} \Gamma^A) \\ &\quad - \frac{1}{288} (\Gamma_{A_1 A_2} \theta) [\bar{\theta} \Gamma^{A_1 A_2 B_1 \dots B_4} F_{B_1 \dots B_4} + 24 \bar{\theta} \Gamma_{B_1 B_2} F^{A_1 A_2 B_1 B_2}], \\ T_A^{B_1 \dots B_4} &= \frac{1}{288} (\Gamma_A^{B_1 \dots B_4} - 8 \delta_A^{[B_1} \Gamma^{B_2 B_3 B_4]}), \quad e^A = dX^M e_M^A, \quad \omega^{AB} = dX^M \omega_M^{AB}, \\ F_{M_1 \dots M_4} &= 6f(\det e_M^A) \epsilon_{M_1 \dots M_4}. \end{aligned}$$

Here  $e_M^A$  and  $\omega_M^{AB}$  are the vielbein and the spin connection, respectively. A constant parameter  $f$  characterizes the  $\text{AdS}_{4/7} \times S^{4/7}$  and is pure imaginary/real. When we take the limit  $f \rightarrow 0$ , the  $\text{AdS}_{4/7} \times S^{7/4}$  backgrounds reduce to flat Minkowski spacetime. So far we have not distinguished closed and open supermembrane, but we will next consider open supermembrane by imposing boundary conditions.

## 2.2 Classification of Dirichlet Branes in $AdS_{4/7} \times S^{7/4}$

In order to make the present paper self-contained, we shall briefly review the classification of Dirichlet branes of an open supermembrane on the  $AdS_{4/7} \times S^{7/4}$  [24] given at the fourth order of  $\theta$ .

Possible Dirichlet branes can be classified by examining the  $\kappa$ -invariance of the covariant open supermembrane. The covariant supermembrane action is invariant under the  $\kappa$ -variation defined as

$$\delta_\kappa E^A \equiv \delta_\kappa Z^{\widehat{M}} E_{\widehat{M}}^A = 0 \quad \rightarrow \quad \delta_\kappa X^M e_M^A = i\bar{\theta}\Gamma^A \delta_\kappa \theta + \mathcal{O}(\theta^4).$$

In the open membrane case, the  $\kappa$ -transformation of the action leads to surface terms and so we need to impose boundary conditions on the boundary of the open membrane world-volume.

The covariant action is written as  $S = \int (\mathcal{L}_{\text{NG}} + \mathcal{L}_{\text{WZ}})$ ,  $\mathcal{L}_{\text{WZ}} = \mathcal{L}_{\text{WZ}}^{\text{bose}} + \mathcal{L}_{\text{WZ}}^0$ . When we perform the  $\kappa$ -transformation, the surface terms appear from the  $\mathcal{L}_{\text{WZ}}^0$  only. The Nambu-Goto part  $\mathcal{L}_{\text{NG}}$  does not contribute to the surface term, because

$$\delta_\kappa \mathcal{L}_{\text{NG}} = \cdots \delta_\kappa E_i^A = \cdots \partial_i (\delta_\kappa Z^{\widehat{M}}) E_{\widehat{M}}^A \sim \partial_i (\cdots \delta_\kappa Z^{\widehat{M}} E_{\widehat{M}}^A) = 0.$$

Moreover, we can make the surface term of the variation  $\delta_\kappa \mathcal{L}_{\text{WZ}}^{\text{bose}}$  vanish by having additional gauge degrees of freedom at the boundary.

It is the place to introduce boundary conditions. In the case of Dirichlet  $p$ -branes, boundary conditions for bosonic variables are usual Neumann and Dirichlet conditions described as

$$\text{Neumann condition :} \quad \overline{A} = A_0, \dots, A_p, \quad \partial_{\mathbf{n}} X^M e_M^{\overline{A}} = 0, \quad (2.7)$$

$$\text{Dirichlet condition :} \quad \underline{A} = A_{p+1}, \dots, A_{10}, \quad \partial_{\mathbf{t}} X^M e_M^{\underline{A}} = 0, \quad (2.8)$$

where  $\partial_{\mathbf{t}}$  and  $\partial_{\mathbf{n}}$  are derivatives with respect to tangential and normal directions, respectively. It is also necessary to take the boundary condition for fermionic variable  $\theta$  as

$$P^+ \theta|_{\partial\Sigma} = 0 \quad \text{or} \quad P^- \theta|_{\partial\Sigma} = 0.$$

Here we have introduced projection operators:

$$P^\pm \equiv \frac{1}{2} (1 \pm M^{10-p}), \quad M^{10-p} \equiv \Gamma^{A_{p+1}} \Gamma^{A_{p+2}} \cdots \Gamma^{A_{10}}.$$

The requirement that  $P^\pm$  are projection operators restricts the value of  $p$  as  $p = 1, 2 \bmod 4$ .

$p = 1$	$p = 5$	$p = 9$
$(1, 1)$	$(3, 3), (1, 5)$	$(3, 7)$

**Tab. 1:** Classification of 1/2 SUSY Dirichlet branes sitting at the origin.

Now let us overview our previous analysis and the result at the fourth order of  $\theta$ . First, in order to ensure the  $\kappa$ -symmetry, we need the vanishing condition

$$\bar{\theta}\Gamma_{\overline{AB}}\delta_{\kappa}\theta = 0. \quad (2.9)$$

This condition restricts the dimension of Dirichlet  $p$ -branes to be  $p = 1, 5, 9$  only. Thus, we have rederived the well-known result in flat space [4, 5].

Second, we have the conditions to preserve the  $\kappa$ -symmetry intrinsic to the AdS geometry:

$$\bar{\theta}\Gamma_{\overline{AB}}T_C^{abcd}\theta = \bar{\theta}\Gamma_{\overline{B}}^CT^{abcd}\theta = 0.$$

This condition leads to the further restriction for the possible configurations of Dirichlet branes. As a result, the world-volume directions of Dirichlet branes are constrained so that the number of Neumann directions in  $\text{AdS}_4$  ( $S^4$ ) of  $\text{AdS}_4 \times S^7$  ( $\text{AdS}_7 \times S^4$ ) is odd. That is,

$$(a, b, c, d) \in \text{AdS}_4(S^4) \quad \sim \quad \{N, D, D, D\} \quad \text{or} \quad \{D, N, N, N\}.$$

The additional surface term coming from the  $\kappa$ -variation of the Wess-Zumino term including  $\mathcal{M}^2$  leads to the condition:

$$\bar{\theta}\Gamma_{\overline{AB}}\mathcal{M}^2\delta_{\kappa}\theta = 0.$$

But this condition is satisfied under the above condition. Moreover, the surface term also appears from the  $\kappa$ -variation of the Wess-Zumino term including spin connections, and gives the conditions:

$$\bar{\theta}\Gamma_{\overline{AB}}\Gamma_{DE}\theta \, \omega_C^{DE} = 0, \quad \bar{\theta}\Gamma_{\overline{B}}^C\Gamma_{DE}\theta \, \omega_B^{DE} = 0.$$

These are satisfied for branes sitting at the origin (i.e.,  $X^{\underline{A}} = 0$ ). It should be noted that no Dirichlet brane can exist outside the origin.

When we use  $(m, n)$  to express the number of Neumann directions in  $(\text{AdS}_4, S^7)$  or  $(S^4, \text{AdS}_7)$ , the possible configurations of 1/2 SUSY D-branes sitting at the origin are summarized in Tab. 1. This result is based on the fourth order analysis with respect to  $\theta$ , but we will see that it is still valid even at full order in the next section.

### 3 Validity of the Classification at Full Order of $\theta$

In the previous section, we have reviewed the classification of possible 1/2 SUSY Dirichlet brane of an open supermembrane on the  $\text{AdS}_{4/7} \times S^{4/7}$  at the fourth order analysis of  $\theta$ . In this section we will show that the classification is still valid even at full order of  $\theta$ . The full order proof for the covariant analysis of D-branes of an AdS-string has been given in [31]. The method used for this proof is also applicable to the open supermembrane case.

The full order proof consists of two important key relations which hold under the 1/2 SUSY conditions obtained at the fourth order analysis. These conditions ensure that higher order terms do not affect the fourth order result. That is, the contribution from higher order terms in  $\theta$  should vanish under the 1/2 SUSY conditions.

Now let us begin with the surface term under the  $\kappa$ -variation at full order of  $\theta$ :

$$\int_{\partial\Sigma} i \int_0^1 dt \widehat{E}_{\mathbf{t}_1}^A \widehat{E}_{\mathbf{t}_2}^B \bar{\theta} \Gamma_{AB} \delta_\kappa \widehat{E}, \quad \widehat{E}^A = E^A(X, t\theta), \quad \widehat{E}^{\bar{\alpha}} = E^{\bar{\alpha}}(X, t\theta), \quad (3.1)$$

where we note that  $\delta_\kappa E_{\mathbf{t}}^A$  produces no surface term. We take  $\theta = P^+ \theta$  as the fermionic boundary condition below, for concreteness. This surface term vanishes under two relations:

$$E_{\mathbf{t}}^A = \partial_{\mathbf{t}} Z^{\widehat{M}} E_{\widehat{M}}^A = 0, \quad (3.2)$$

$$P^- \delta_\kappa E^\alpha = P^- \delta_\kappa Z^{\widehat{M}} E_{\widehat{M}}^\alpha = 0, \quad (3.3)$$

because we can rewrite the full order surface term (3.1) as follows:

$$(3.1) = \int_{\partial\Sigma} i \int_0^1 dt \widehat{E}_{\mathbf{t}_1}^{\bar{A}} \widehat{E}_{\mathbf{t}_2}^{\bar{B}} \bar{\theta} P^- \Gamma_{\bar{A}\bar{B}} P^- \delta_\kappa \widehat{E} = 0.$$

It should be noted that the  $t$ -integral contributes to a numerical coefficient of each term in the Wess-Zumino term and it does not affect our consideration below.

We will prove the important equations (3.2) and (3.3) under the 1/2 SUSY conditions.

#### 3.1 Proof of (3.2)

Let us show that (3.2) should hold under the 1/2 SUSY conditions for Dirichlet  $p$ -branes ( $p = 1, 5$  and  $9$ ).

First, we will show the equation,

$$\partial_{\mathbf{t}} X^M E_M^A = 0. \quad (3.4)$$

The l. h. s. of (3.4) can be rewritten as

$$\begin{aligned}\partial_{\mathbf{t}} X^M E_M^A &= \partial_{\mathbf{t}} X^M \left[ -i\bar{\theta} \Gamma^A \left( \frac{2}{\mathcal{M}} \sinh \frac{\mathcal{M}}{2} \right)^2 [D\theta]_M \right] \\ &= -i\partial_{\mathbf{t}} X^M \bar{\theta} P^- \Gamma^A P^- \left( \frac{2}{\mathcal{M}} \sinh \frac{\mathcal{M}}{2} \right)^2 P^- [D\theta]_M,\end{aligned}\quad (3.5)$$

where we have used Dirichlet condition (2.8),  $\partial_{\mathbf{t}} X^M e_M^A = 0$  in the first equality, and the identity

$$P^- \left( \frac{2}{\mathcal{M}} \sinh \frac{\mathcal{M}}{2} \right)^2 P^+ = 0 \quad (3.6)$$

in the last equality. The relation (3.6) follows from

$$\begin{aligned}P^- i\mathcal{M}^2 &= 2P^- T_{\underline{A}}^{a_1 \dots a_4} P^+ \theta F_{a_1 \dots a_4} \bar{\theta} P^- \Gamma^{\underline{A}} P^- \\ &\quad - \frac{1}{288} P^- \Gamma_{\underline{A_1 A_2}} P^+ \theta \bar{\theta} P^- \Gamma^{\underline{A_1 A_2}} P^- F_{a_1 \dots a_4} \\ &\quad - \frac{1}{288} P^- \Gamma_{\underline{a_1 a_2}} P^+ \theta \bar{\theta} P^- 24 \Gamma_{a_3 a_4} P^- F^{\underline{a_1 a_2} a_3 a_4} \\ &= P^- i\mathcal{M}^2 P^-, \end{aligned}\quad (3.7)$$

where the number of Neumann directions in  $\{a_1, \dots, a_4\}$  is odd. We notice that (3.5) vanishes if

$$\partial_{\mathbf{t}} X^M P^- [D\theta]_M = 0. \quad (3.8)$$

The relation (3.8) can be shown as follows

$$\begin{aligned}\partial_{\mathbf{t}} X^M P^- [D\theta]_M &= \partial_{\mathbf{t}} X^M e_M^A P^- T_{\underline{A}}^{a_1 \dots a_4} P^+ \theta F_{a_1 \dots a_4} - \frac{1}{4} \partial_{\mathbf{t}} X^M e_M^{\underline{B}} \omega_{\underline{B}}^{\underline{A_1 A_2}} P^- \Gamma_{\underline{A_1 A_2}} P^+ \theta \\ &= 0,\end{aligned}\quad (3.9)$$

because we have two relations:  $\partial_{\mathbf{t}} X^M e_M^A = 0$  from the Dirichlet condition and the vanishing spin connection  $\omega_{\underline{B}}^{\underline{A_1 A_2}} = 0$  at the origin. Thus, we have shown that the relation (3.4) is satisfied under the 1/2 SUSY conditions.

In addition, we can easily see that  $\partial_{\mathbf{t}} \theta^\alpha E_\alpha^A = 0$  as follows:

$$\partial_{\mathbf{t}} \theta^\alpha E_\alpha^A = i\bar{\theta} P^- \Gamma^A P^- \left( \frac{2}{\mathcal{M}} \sinh \frac{\mathcal{M}}{2} \right)^2 P^- \partial_{\mathbf{t}} \theta = 0. \quad (3.10)$$

Plugging (3.4) with (3.10), we obtain that

$$E_{\mathbf{t}}^A = \partial_{\mathbf{t}} \widehat{Z^M} E_M^A = \partial_{\mathbf{t}} X^M E_M^A + \partial_{\mathbf{t}} \theta^\alpha E_\alpha^A = 0,$$



and so the relation (3.2) has been proven.

As a final remark in this subsection, we should note that the relation (3.2) means that Dirichlet branes are static. This fact is consistent to the 1/2 SUSY conditions. Next, we will show the second relation (3.3).

### 3.2 Proof of (3.3)

Here we will show the relation (3.3) under the 1/2 SUSY conditions.

First of all, we will show the relation,

$$\delta_\kappa X^M e_M^A = 0. \quad (3.11)$$

The definition of  $\kappa$ -variation,

$$\delta_\kappa E^A = \delta_\kappa \widehat{Z^M} E_M^A = 0 \quad (3.12)$$

implies that

$$\delta x^A = H^A_B \delta x^B + \delta \theta^A, \quad (3.13)$$

where

$$\delta x^A \equiv \delta_\kappa X^M e_M^A, \quad \delta \theta^A = -\delta_\kappa \theta^\alpha E_\alpha^A, \quad H^A_B = i\bar{\theta}\Gamma^A \left( \frac{2}{\mathcal{M}} \sinh \frac{\mathcal{M}}{2} \right)^2 [D\theta]_M e_B^M. \quad (3.14)$$

The recursive relation (3.13) leads to the following expression:

$$\delta x^A = (1 + H + H^2 + \dots + H^{15})^A_B \delta \theta^B, \quad (3.15)$$

so that we have

$$\delta_\kappa X^M e_M^A = -i(1 + H + H^2 + \dots + H^{15})^A_{\bar{B}} \bar{\theta} P^- \Gamma^{\bar{B}} P^- \left( \frac{2}{\mathcal{M}} \sinh \frac{\mathcal{M}}{2} \right)^2 P^+ \delta_\kappa \theta. \quad (3.16)$$

The relation (3.11) is satisfied when  $H^A_{\bar{B}} = 0$ . In fact, we can easily see that  $H^A_{\bar{B}} = 0$  as follows:

$$H^A_{\bar{B}} = i\bar{\theta}\Gamma^A P^- \left( \frac{2}{\mathcal{M}} \sinh \frac{\mathcal{M}}{2} \right)^2 P^- [D\theta]_M e_{\bar{B}}^M = 0. \quad (3.17)$$

Here we have used the following relation:

$$P^- [D\theta]_M e_{\bar{B}}^M = P^- T_{\bar{B}}^{a_1 \dots a_4} P^- \theta F_{a_1 \dots a_4} - \frac{1}{4} \omega_{\bar{B}}^{\bar{A}_1 \bar{A}_2} P^- \Gamma_{\bar{A}_1 \bar{A}_2} P^+ \theta = 0. \quad (3.18)$$

Thus, we have shown that (3.11) is satisfied.

Next we shall prove

$$P^- \delta_\kappa X^M E_M^{\bar{\alpha}} = P^- \frac{\sinh \mathcal{M}}{\mathcal{M}} P^- [D\theta]_M \delta_\kappa X^M = 0. \quad (3.19)$$

This relation (3.19) follows from

$$\begin{aligned} P^- [D\theta]_M \delta_\kappa X^M &= e_M^A P^- T_A^{a_1 \dots a_4} P^+ \theta F_{a_1 \dots a_4} \delta_\kappa X^M - \frac{1}{4} \omega_{\bar{B}}^{\bar{A}_1 A_2} e_M^{\bar{B}} P^- \Gamma_{\bar{A}_1 \bar{A}_2} P^+ \theta \delta_\kappa X^M \\ &= 0, \end{aligned} \quad (3.20)$$

where we have used (3.11) and the vanishing spin connection  $\omega_{\bar{B}}^{\bar{A}_1 A_2} = 0$  at the origin.

Furthermore, we can show that

$$P^- \delta_\kappa \theta^\beta E_\beta^{\bar{\alpha}} = -P^- \frac{\sinh \mathcal{M}}{\mathcal{M}} P^+ \delta_\kappa \theta = 0. \quad (3.21)$$

Combining (3.11) and (3.21), we find that

$$P^- \delta_\kappa Z^{\widehat{M}} E_{\widehat{M}}^{\bar{\alpha}} = P^- \delta_\kappa X^M E_M^{\bar{\alpha}} + P^- \delta_\kappa \theta^\beta E_\beta^{\bar{\alpha}} = 0, \quad (3.22)$$

and so we have finished the proof of (3.3).

Thus, we have shown two key relations (3.2) and (3.3). Namely, the classification of 1/2 SUSY Dirichlet branes on the  $\text{AdS}_{4/7} \times S^{7/4}$  has been completed in the present.

### 3.3 Validity of the Classification in PP-wave Case

Here we shall consider the Dirichlet branes of an open supermembrane on the maximally supersymmetric pp-wave background. These are discussed in our previous work [21], and the possible configurations of 1/2 SUSY Dirichlet branes sitting at and outside the origin are summarized in Tabs. 2 and 3, respectively. While the pp-wave background is obtained from the  $\text{AdS}_{4/7} \times S^{7/4}$  via a Penrose limit [25, 26], the classification of the allowed Dirichlet branes on the pp-wave is realized from that on the  $\text{AdS}_{4/7} \times S^{7/4}$ . The classification result in the pp-wave case is also based on the fourth order analysis with respect to  $\theta$ . Then we will prove that it is also valid at full order of  $\theta$ .

The scenario of the full order proof is almost the same as in the case of  $\text{AdS}_{4/7} \times S^{7/4}$ . So, we do not carry out the proof explicitly, but only the difference between the  $\text{AdS}_{4/7} \times S^{7/4}$  and pp-wave cases should be explained. The difference is as follows:

	N: +, -	D: +, -
$p = 9$	$(+, -; 2, 6)$	
$p = 5$	$(+, -; 0, 4), (+, -; 2, 2)$	$(1, 5), (3, 3)$
$p = 1$	$(+, -; 0, 0)$	$(1, 1)$

**Tab. 2:** Classification of 1/2 SUSY Dirichlet branes sitting at the origin (pp-wave case).

	N: +, -	D: +, -
$p = 9$		
$p = 5$		$(1, 5), (3, 3)$
$p = 1$	$(+, -; 0, 0)$	$(1, 1)$

**Tab. 3:** Classification of 1/2 SUSY Dirichlet branes sitting outside the origin (pp-wave case).

- The indices  $(a_1, \dots, a_4)$  in the  $\text{AdS}_{4/7} \times S^{7/4}$  are replaced by  $(+, 1, 2, 3)$ .
- The spin connection in the  $\text{AdS}_{4/7} \times S^{7/4}$  is replaced by that in the pp-wave. The spin connection of pp-wave is quite simple and almost zero. The only non-vanishing component of  $\omega_M^{AB}$  is given by

$$\omega_+^{r-} = \frac{1}{2} \partial^r G_{++} \quad (r = 1, \dots, 9), \quad \text{others} = 0,$$

where  $G_{++}$  is the  $(+, +)$ -component of the pp-wave metric in Brinkmann coordinates represented by

$$G_{++} = - \left[ \left( \frac{\mu}{3} \right)^2 (X_1^2 + X_2^2 + X_3^2) + \left( \frac{\mu}{6} \right)^2 (X_4^2 + \dots + X_9^2) \right].$$

First, we note that the first difference does not change the scenario of the proof. Then the replacement of the spin connection also makes no change at the origin because the spin connections in both  $\text{AdS}_{4/7} \times S^{7/4}$  and pp-wave vanish at the origin. The main difference appears in the analysis outside the origin. In the pp-wave case, the spin connection is almost zero and the non-vanishing component is  $\omega_+^{r-}$  only. Hence the spin connection does not so severely restrict the brane configuration outside the origin. In fact, we found that several 1/2 SUSY Dirichlet branes can exist outside the origin in comparison to the  $\text{AdS}_{4/7} \times S^{7/4}$  cases.

So, from now on, let us focus on the validity of 1/2 SUSY Dirichlet branes sitting outside the origin. In the case that both  $+-$  and  $--$ -directions satisfy the Dirichlet condition, the

condition  $\omega_{\overline{B}}^{\overline{A_1 A_2}} = 0$  is trivially satisfied since the  $+$ -direction is a Dirichlet one. The remaining part we need to consider is a configuration of  $(+, -; 0, 0)$ . The condition  $\Gamma^{1 \cdots 9} \theta = \theta$  leads to  $\Gamma^{+-} \theta = \theta$  via the relation  $\Gamma^{01 \cdots 910} = \mathbb{I}_{32}$ . Then we obtain the relation  $\frac{1}{2} \Gamma_- \Gamma_+ \theta = \theta$  by using  $-2\mathbb{I}_{32} = \Gamma_+ \Gamma_- + \Gamma_- \Gamma_+$ , so that  $\Gamma_- \theta = 0$ . It follows that equations (3.9), (3.18) and (3.20) are satisfied by the  $(+, -)$ -string sitting outside the origin. Hence the configuration of  $(+, -)$ -string sitting outside the origin is allowed even at full order of  $\theta$ . Thus, we have finished the full order proof for the pp-wave case, and the classification of possible configurations of 1/2 SUSY Dirichlet branes on the pp-wave background is valid at full order of  $\theta$ , in spite of the existence of several configurations sitting outside the origin.

## 4 Open M5-brane on $\text{AdS}_{4/7} \times \mathbf{S}^{7/4}$

In this section we shall consider an open M5-brane on the  $\text{AdS}_{4/7} \times S^{7/4}$ , according to our method used in the open M2-brane case. To begin with, we introduce the covariant action of M5-brane [32] on the  $\text{AdS}_{4/7} \times S^{7/4}$  constructed in [33]. Then we discuss Dirichlet branes of the M5-brane, by following the scenario employed in the M2-brane case.

### 4.1 Covariant Super-M5-Brane on $\text{AdS}_{4/7} \times S^{7/4}$

The covariant M5-brane action in flat space was proposed by Pasti-Sorokin-Tonin (PST) [32]. Then Claus constructed the PST action of an M5-brane on the  $\text{AdS}_{4/7} \times S^{7/4}$  [33]. The covariant action of M5-brane consists of two parts as

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{WZ}}. \quad (4.1)$$

The  $\mathcal{L}_0$  part is Dirac-Born-Infeld (DBI) type action:

$$\begin{aligned} \mathcal{L}_0 &= \sqrt{-\det(g_{ij} + i\mathcal{H}_{ij}^*)} + \frac{\sqrt{-g}}{4} \mathcal{H}^{*ij} \mathcal{H}_{ij}, \\ \mathcal{H}_{ij} &= \mathcal{H}_{ijk} v^k, \quad \mathcal{H}_{ij}^* = \mathcal{H}_{ijk}^* v^k, \quad v_i = \frac{\partial_i a}{\sqrt{g^{jk} \partial_j a \partial_k a}}, \\ \mathcal{H} &= H + A_3, \quad \mathcal{H}_{ijk}^* = \frac{\sqrt{g}}{3!} \epsilon_{ijklmn} \mathcal{H}^{lmn}, \quad H = dB. \end{aligned} \quad (4.2)$$

Here the PST scalar field  $a$  is contained in the M5-brane case as a modification of the usual DBI action. The  $\mathcal{L}_{\text{WZ}}$  part is the Wess-Zumino term

$$\mathcal{L}_{\text{WZ}} = A_6 + \frac{1}{2}H \wedge A_3, \quad (4.3)$$

$$A_3 = i \int_0^1 dt \hat{E}^A \hat{E}^B \bar{\theta} \Gamma_{AB} \hat{E} + C_3, \quad (4.4)$$

$$A_6 = i \int_0^1 dt \left( \frac{2}{5!} \hat{E}^{A_1} \dots \hat{E}^{A_5} \bar{\theta} \Gamma_{A_1 \dots A_5} \hat{E} + \frac{1}{2} \hat{A}_3 \wedge \hat{E}^A \hat{E}^B \bar{\theta} \Gamma_{AB} \hat{E} \right) + C_6, \quad (4.5)$$

where  $C_3$  and  $C_6$  are bosonic terms defined by  $C_3 = \frac{1}{3!} e^A \wedge e^B \wedge e^C C_{ABC}$  and  $C_6 = \frac{1}{6!} e^{A_1} \wedge \dots \wedge e^{A_6} C_{A_1 \dots A_6}$ . The Wess-Zumino term  $\mathcal{L}_{\text{WZ}}$  can be expressed as

$$i \int_0^1 dt \left( \frac{2}{5!} \hat{E}^{A_1} \dots \hat{E}^{A_5} \bar{\theta} \Gamma_{A_1 \dots A_5} \hat{E} + \frac{1}{2} \hat{\mathcal{H}} \wedge \hat{E}^A \hat{E}^B \bar{\theta} \Gamma_{AB} \hat{E} \right) + C_6 + \frac{1}{2} H \wedge C_3, \quad (4.6)$$

where the symbols with “hat” implies that the fermionic variable  $\theta$  is rescaled as  $\theta \rightarrow t\theta$ . Here we have used the fact that  $H$  does not depend on  $\theta$  because  $H$  is the world-volume gauge field strength independent of  $X$  and  $\theta$ . Next, we will consider the Dirichlet branes in the next subsection.

#### 4.2 Dirichlet brane of M5-brane on $\text{AdS}_{4/7} \times S^{7/4}$

Now we shall consider Dirichlet branes of an open M5-brane on the  $\text{AdS}_{4/7} \times S^{7/4}$  by examining the  $\kappa$ -variation of the action  $\mathcal{L}$ . The definition of the  $\kappa$ -variation of  $a$  is  $\delta_\kappa a = 0$ , which implies that the  $\kappa$ -variation surface term of  $v_i$  vanishes because  $\delta_\kappa v_i = \dots \delta_\kappa g_{ij}$ . On the other hand, the  $\kappa$ -variation of  $\mathcal{H}_{ijk}$  produces no surface term because the  $\kappa$ -variation of  $B$  is defined so as to absorb the  $\kappa$ -variation surface term of  $A_3$  and thus  $\delta_\kappa \mathcal{H}_{ijk}$  includes no surface term. Hence,  $\delta_\kappa \mathcal{L}_0$  does not contribute to the  $\kappa$ -variation surface term.

The  $\kappa$ -variation surface term of  $\mathcal{L}$  comes from  $\mathcal{L}_{\text{WZ}}$  in (4.6). The contribution from the bosonic terms can be deleted by having additional degrees of freedom on the boundary. Noting that  $\delta_\kappa \mathcal{H}$  produces no surface term and the surface term of  $\delta_\kappa E^A$  vanishes, the surface term takes the form:

$$i \int_0^1 dt \epsilon^{t_1 \dots t_5} \left( \frac{2}{5!} \hat{E}_{t_1}^{A_1} \dots \hat{E}_{t_5}^{A_5} \bar{\theta} \Gamma_{A_1 \dots A_5} \delta_\kappa \hat{E} + \frac{1}{2} \hat{\mathcal{H}}_{t_1 t_2 t_3} \hat{E}_{t_4}^A \hat{E}_{t_5}^B \bar{\theta} \Gamma_{AB} \delta_\kappa \hat{E} \right). \quad (4.7)$$

The vanishing condition for the second term is that

$$E_{t_4}^A E_{t_5}^B \bar{\theta} \Gamma_{AB} \delta_\kappa Z^{\widehat{M}} E_{\widehat{M}} = 0, \quad (4.8)$$

for a generic  $H$  at the boundary. This condition is nothing but the vanishing condition examined in the last section. If we can take the vanishing configuration  $H = 0$  at the boundary, an alternative condition is that

$$E_{t_1}^A E_{t_2}^B \bar{\theta} \Gamma_{AB} \partial_{t_3} Z^{\widehat{M}} E_{\widehat{M}} = 0, \quad (4.9)$$

and this condition leads to the same brane configurations derived from the condition (4.8). Thus the Dirichlet brane configurations of an M5-brane must lie in that of an M2-brane obtained before.

Under the 1/2 SUSY conditions examined in the last section, the conditions to vanish the surface term (4.7) are

$$\bar{\theta} \Gamma_{\overline{A_1} \dots \overline{A_5}} \delta_{\kappa} Z^{\widehat{M}} E_{\widehat{M}} = 0. \quad (4.10)$$

However this condition is not satisfied

$$\bar{\theta} P^- \Gamma_{\overline{A_1} \dots \overline{A_5}} P^+ \delta_{\kappa} Z^{\widehat{M}} E_{\widehat{M}} \neq 0. \quad (4.11)$$

Thus, we find that there is no Dirichlet brane of an open M5-brane. We have considered here an open M5-brane on the  $\text{AdS}_{4/7} \times S^{7/4}$ , but this is the case even in flat space, as one can easily see by considering the flat limit  $f \rightarrow 0$ .

## 5 Conclusion and Discussion

We have promoted a study of an open supermembrane on the  $\text{AdS}_{4/7} \times S^{7/4}$  backgrounds. The classification of 1/2 SUSY Dirichlet branes, given at the fourth order analysis with respect to  $\theta$ , has been shown to be valid even at full order of  $\theta$ . The full order proof here supplements our previous result. The classification of 1/2 SUSY Dirichlet branes on the pp-wave background, which is given at the fourth order of  $\theta$  in [21], has been also revisited and has been shown to be valid at full order of  $\theta$  in the same way as the  $\text{AdS}_{4/7} \times S^{7/4}$  cases, in spite of the existence of several configurations sitting outside the origin.

In addition we have considered Dirichlet branes of an open M5-brane on the  $\text{AdS}_{4/7} \times S^{7/4}$ . By using the PST action of the M5-brane and following the M2-brane analysis, it has been shown that the open M5-brane cannot have 1/2 SUSY Dirichlet branes. This is the case even in flat space.

It would be also interesting to consider D-brane with boundary (i.e., open D-brane) in type IIB string theory on the  $AdS_5 \times S^5$ , according to our method while we have discussed open M5-brane in this paper. We will report on this issue in the near future [34].

## Acknowledgments

The work of M. S. is supported by the 21 COE program “Constitution of wide-angle mathematical basis focused on knots”. The work of K. Y. is supported in part by JSPS Research Fellowships for Young Scientists.

## Appendix

### Notation and Convention

In this place we will summarize miscellaneous notation and convention used in this paper.

#### *Notation of Coordinates*

For the supermembrane in the eleven-dimensional curved space-time, we use the following notation of supercoordinates for its superspace:

$$(X^M, \theta^\alpha), \quad M = (\mu, \mu'), \quad \mu \in AdS_4(S^4), \quad \mu' \in S^7(AdS_7),$$

and the background metric is expressed by  $G_{MN}$ . The coordinates in the Lorentz frame is denoted by

$$(X^A, \theta^{\bar{\alpha}}), \quad A = (a, a'), \quad a = \begin{cases} 0, 1, 2, 3 \\ 10, 1, 2, 3 \end{cases}, \quad a' = \begin{cases} 4, \dots, 9, 10 & \text{for } AdS_4 \times S^7 \\ 0, 4, \dots, 9 & \text{for } AdS_7 \times S^4 \end{cases},$$

and its metric is described by  $\eta_{AB} = \text{diag}(-1, +1, \dots, +1)$  with  $\eta_{00} = -1$ .

The membrane world-volume is three-dimensional and its coordinates are parameterized by  $(\tau, \sigma^1, \sigma^2)$ . The induced metric on the world-volume is represented by  $g_{ij}$ .

## *SO(10, 1) Clifford Algebra*

We denote a 32-component Majorana spinor as  $\theta$ , and the  $SO(10, 1)$  gamma matrices  $\Gamma^A$ 's satisfy the  $SO(10, 1)$  Clifford algebra

$$\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}, \quad \{\Gamma^M, \Gamma^N\} = 2G^{MN}, \quad \Gamma^A \equiv e_M^A \Gamma^M, \quad \Gamma^M \equiv e_A^M \Gamma^A,$$

where the light-cone components of the  $SO(10, 1)$  gamma matrices are

$$\Gamma^\pm \equiv \frac{1}{\sqrt{2}} (\Gamma^0 \pm \Gamma^{10}), \quad \{\Gamma^+, \Gamma^-\} = -2\mathbb{I}_{32}.$$

We shall choose  $\Gamma^A$ 's such that  $\Gamma^0$  is anti-hermite matrix and others are hermite matrices. In this choice the relation  $(\Gamma^A)^\dagger = \Gamma_0 \Gamma^A \Gamma_0$  is satisfied. The charge conjugation of  $\theta$  is defined by

$$\theta^c \equiv \mathcal{C} \bar{\theta}^T,$$

where  $\bar{\theta}$  is the Dirac conjugation of  $\theta$  and is defined by  $\bar{\theta} \equiv \theta^\dagger \Gamma_0$ . The charge conjugation matrix  $\mathcal{C}$  satisfies the following relation:

$$(\Gamma^A)^T = -\mathcal{C}^{-1} \Gamma^A \mathcal{C}, \quad \mathcal{C}^T = -\mathcal{C}.$$

For an arbitrary Majorana spinor  $\theta$  satisfying the Majorana condition  $\theta^c = \theta$ , we can easily show the formula

$$\bar{\theta} = -\theta^T \mathcal{C}^{-1}.$$

That is, the charge conjugation matrix  $\mathcal{C}$  is defined by  $\mathcal{C} = \Gamma_0$  in this representation. The  $\Gamma^A$ 's are real matrices (i.e.,  $(\Gamma^A)^* = \Gamma^A$ ). We also see that  $\Gamma^r$  ( $r = 1, 2, \dots, 9$ ) and  $\Gamma^{10}$  are symmetric and  $\Gamma^0$  is skewsymmetric.

## References

- [1] E. Bergshoeff, E. Sezgin and P. K. Townsend, ‘‘Supermembranes And Eleven-Dimensional Supergravity,’’ *Phys. Lett. B* **189** (1987) 75; ‘‘Properties Of The Eleven-Dimensional Super Membrane Theory,’’ *Annals Phys.* **185** (1988) 330.
- [2] B. de Wit, J. Hoppe and H. Nicolai, ‘‘On The Quantum Mechanics Of Supermembranes,’’ *Nucl. Phys. B* **305** (1988) 545.



- [3] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, “M theory as a matrix model: A conjecture,” *Phys. Rev. D* **55** (1997) 5112 [arXiv:hep-th/9610043].
- [4] K. Ezawa, Y. Matsuo and K. Murakami, “Matrix regularization of open supermembrane: Towards M-theory five-brane via open supermembrane,” *Phys. Rev. D* **57** (1998) 5118 [arXiv:hep-th/9707200].
- [5] B. de Wit, K. Peeters and J. C. Plefka, “Open and closed supermembranes with winding,” *Nucl. Phys. Proc. Suppl.* **68** (1998) 206 [arXiv:hep-th/9710215].
- [6] P. Horava and E. Witten, “Heterotic and type I string dynamics from eleven dimensions,” *Nucl. Phys. B* **460** (1996) 506 [arXiv:hep-th/9510209]; “Eleven-Dimensional Supergravity on a Manifold with Boundary,” *Nucl. Phys. B* **475** (1996) 94 [arXiv:hep-th/9603142].
- [7] C. S. Chu and E. Sezgin, “M-fivebrane from the open supermembrane,” *JHEP* **9712** (1997) 001 [arXiv:hep-th/9710223]; C. S. Chu, P. S. Howe and E. Sezgin, “Strings and D-branes with boundaries,” *Phys. Lett. B* **428** (1998) 59 [arXiv:hep-th/9801202].
- [8] J. Kowalski-Glikman, “Vacuum States In Supersymmetric Kaluza-Klein Theory,” *Phys. Lett. B* **134** (1984) 194.
- [9] D. Berenstein, J. M. Maldacena and H. Nastase, “Strings in flat space and pp waves from  $N = 4$  super Yang Mills,” *JHEP* **0204** (2002) 013 [arXiv:hep-th/0202021].
- [10] K. Dasgupta, M. M. Sheikh-Jabbari and M. Van Raamsdonk, “Matrix perturbation theory for M-theory on a PP-wave,” *JHEP* **0205** (2002) 056 [arXiv:hep-th/0205185].
- [11] K. Sugiyama and K. Yoshida, “Supermembrane on the pp-wave background,” *Nucl. Phys. B* **644** (2002) 113 [arXiv:hep-th/0206070].
- [12] K. Sugiyama and K. Yoshida, “BPS conditions of supermembrane on the pp-wave,” *Phys. Lett. B* **546** (2002) 143 [arXiv:hep-th/0206132]; N. Nakayama, K. Sugiyama and K. Yoshida, “Ground state of the supermembrane on a pp-wave,” *Phys. Rev. D* **68** (2003) 026001 [arXiv:hep-th/0209081].
- [13] T. Kimura and K. Yoshida, “Spectrum of eleven-dimensional supergravity on a pp-wave background,” *Phys. Rev. D* **68** (2003) 125007 [arXiv:hep-th/0307193].

- [14] N. D. Lambert and P. C. West, “D-branes in the Green-Schwarz formalism,” *Phys. Lett. B* **459** (1999) 515 [arXiv:hep-th/9905031].
- [15] R. R. Metsaev, “Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background,” *Nucl. Phys. B* **625** (2002) 70 [arXiv:hep-th/0112044].
- [16] R. R. Metsaev and A. A. Tseytlin, “Exactly solvable model of superstring in plane wave Ramond-Ramond background,” *Phys. Rev. D* **65** (2002) 126004 [arXiv:hep-th/0202109].
- [17] K. Sugiyama and K. Yoshida, “Type IIA string and matrix string on pp-wave,” *Nucl. Phys. B* **644** (2002) 128 [arXiv:hep-th/0208029]; S. Hyun and H. Shin, “ $N = (4,4)$  type IIA string theory on pp-wave background,” *JHEP* **0210** (2002) 070 [arXiv:hep-th/0208074].
- [18] P. Bain, K. Peeters and M. Zamaklar, “D-branes in a plane wave from covariant open strings,” *Phys. Rev. D* **67** (2003) 066001 [arXiv:hep-th/0208038].
- [19] S. Hyun, J. Park and H. Shin, “Covariant description of D-branes in IIA plane-wave background,” *Phys. Lett. B* **559** (2003) 80 [arXiv:hep-th/0212343].
- [20] S. Hyun and H. Shin, “Solvable  $N = (4,4)$  type IIA string theory in plane-wave background and D-branes,” *Nucl. Phys. B* **654** (2003) 114 [arXiv:hep-th/0210158].  
H. Shin, K. Sugiyama and K. Yoshida, “Partition function and open/closed string duality in type IIA string theory on a pp-wave,” *Nucl. Phys. B* **669** (2003) 78 [arXiv:hep-th/0306087].  
Y. Kim and J. Park, “Boundary states in IIA plane-wave background,” *Phys. Lett. B* **572** (2003) 81 [arXiv:hep-th/0306282].
- [21] M. Sakaguchi and K. Yoshida, “Dirichlet branes of the covariant open supermembrane on a pp-wave background,” *Nucl. Phys. B* **676** (2004) 311 [arXiv:hep-th/0306213].
- [22] L. Motl, A. Neitzke and M. M. Sheikh-Jabbari, “Heterotic plane wave matrix models and giant gluons,” arXiv:hep-th/0306051.
- [23] B. de Wit, K. Peeters, J. Plefka and A. Sevrin, “The M-theory two-brane in  $AdS(4) \times S(7)$  and  $AdS(7) \times S(4)$ ,” *Phys. Lett. B* **443** (1998) 153 [arXiv:hep-th/9808052].
- [24] M. Sakaguchi and K. Yoshida, “Dirichlet branes of the covariant open supermembrane in  $AdS_4 \times S^7$  and  $AdS_7 \times S^4$ ,” *Nucl. Phys. B* **681** (2004) 137 [arXiv:hep-th/0310035].

- [25] R. Penrose, “Any spacetime has a plane wave as a limit,” *Differential geometry and relativity*, Reidel, Dordrecht, 1976, pp. 271-275.  
R. Gueven, “Plane wave limits and T-duality,” *Phys. Lett. B* **482** (2000) 255 [arXiv:hep-th/0005061].
- [26] M. Blau, J. Figueroa-O’Farrill, C. Hull and G. Papadopoulos, “Penrose limits and maximal supersymmetry,” *Class. Quant. Grav.* **19** (2002) L87 [arXiv:hep-th/0201081].
- [27] M. Hatsuda, K. Kamimura and M. Sakaguchi, “Super-PP-wave algebra from super-AdS x S algebras in eleven-dimensions,” *Nucl. Phys. B* **637** (2002) 168 [arXiv:hep-th/0204002];  
“From super-AdS<sub>5</sub> × S<sup>5</sup> algebra to super-pp-wave algebra,” *Nucl. Phys. B* **632** (2002) 114 [arXiv:hep-th/0202190].
- [28] N. Kim and J. T. Yee, “Supersymmetry and branes in M-theory plane-waves,” *Phys. Rev. D* **67** (2003) 046004 [arXiv:hep-th/0211029].
- [29] M. Sakaguchi and K. Yoshida, “D-branes of covariant AdS superstrings,” *Nucl. Phys. B* **684** (2004) 100 [arXiv:hep-th/0310228].
- [30] K. Skenderis and M. Taylor, “Branes in AdS and pp-wave spacetimes,” *JHEP* **0206** (2002) 025 [arXiv:hep-th/0204054].
- [31] M. Sakaguchi and K. Yoshida, “Notes on D-branes of type IIB string on  $AdS_5 \times S^5$ ,” to be published in *Phys. Lett. B*, arXiv:hep-th/0403243.
- [32] P. Pasti, D. P. Sorokin and M. Tonin, “Covariant action for a D = 11 five-brane with the chiral field,” *Phys. Lett. B* **398** (1997) 41 [arXiv:hep-th/9701037].  
I. A. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. P. Sorokin and M. Tonin, “Covariant action for the super-five-brane of M-theory,” *Phys. Rev. Lett.* **78** (1997) 4332 [arXiv:hep-th/9701149].
- [33] P. Claus, “Super M-brane actions in AdS(4) x S(7) and AdS(7) x S(4),” *Phys. Rev. D* **59** (1999) 066003 [arXiv:hep-th/9809045].
- [34] M. Sakaguchi and K. Yoshida, in preparation.